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## A New Approach to Performance Analysis of Disinfection Using Grey System Theory

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### Abstract

It is important to reduce sanitary risks related to the presence of pathogens and disinfection can be achieved via chlorination. In this paper, GM (1, 1) model was employed to predict the rate of inactivation with free chlorine for *Enterococcus faecalis*. The result indicated that GM only required a small amount of data can predict well. The mean absolute percentage errors (MAPEs) using GM(1,1) model lay between 0.04%-9.75%. Research results show that this GM(1,1) model can provide valuable information for public health.

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**Keywords:** *Enterococcus faecalis*; GM(1,1) model; inactivation; grey system theory

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### 1. Introduction

Potable water is a probable source for infection. The main objective of disinfection is to reduce sanitary risks related to the presence of pathogens. Water disinfection can be achieved via different means such as chlorination, ozonation, ultraviolet radiation and the effect of chlorine dioxide<sup>[1-6]</sup>; Chlorination is widely used as a disinfection process for bacteria control in water. When properly designed and operated, the process is well-developed, cheap, and impactful, despite the major problem of harmful disinfection by-products generated by this treatment<sup>[7-10]</sup>.

*Enterococcus faecalis* are normal flora and gram-positive bacteria which presence both human and animal's stomach and intestine. It has been listed in the appendices of Standards for Drinking Water Quality in China<sup>[11]</sup>. The inactivation of *Enterococcus faecalis* by free chlorine has not been characterized previously in any water quality conditions. And it was used as a representative bacteria in the current study due to its relatively robust viability assay than the *E.coli*.

Different models have been developed to study the influence of the value of Ct on disinfection<sup>[12,13]</sup>. They were founded by using regression analysis and artificial neural network(ANN). Although these methods could predict the result successfully, a large quantity of data should be screened in advance for further calculation. In order to gain consistent results from the testing data and predict the complete disinfection trend, the grey system theory (GST) is an appropriate method. It has been used in many fields and has got an obvious achievement<sup>[14-16]</sup>. But few research has been reported on disinfection using this method. The purpose of this study was to characterize the effects of the value of Ct on the kinetics of *Enterococcus faecalis* inactivation with free chlorine using grey system theory.

## 2. Grey Modeling Process

One can create a GM to describe the behavior of the few outputs using fewer (at least 4) system's data and uses predictions to analyze future situations. A series of generation and transformation procedures involving secondary data are used to find the internal regularities of grey quantities, which often can not be done employing statistical analysis techniques. By means of accumulated generating operation (AGO), which is one of the most important characteristics of grey theory, the unsystematic data may become exponentially behaved such that a first-order differential equation can be used to characterize the system behavior. Through inverse accumulated generating operation (IAGO), the prediction can be transformed back to the sequence of original series. A grey modeling(GM(1,1)) process is explained as follows.

Assume that the original series of data is expressed as:

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) \quad (1)$$

$X^{(1)}$  be the first-order AGO of  $X^{(0)}$ , whose elements are generated from  $X^{(0)}$ :

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \quad (2)$$

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i) \quad k=1,2,\dots,n; \quad (3)$$

Inverse accumulated generating operation. The mathematical model is shown below:  $x^{(0)}(1) = x^{(1)}(1)$

$$x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1) \quad k=2,3,\dots,n; \quad (4)$$

According to the grey system theory, the GM (1, 1) model can be generally expressed as

$$dx^{(1)} / dt + ax^{(1)} = b$$

where the parameter a is the developing coefficient and b is the grey input.  
we define the background value is

$$z^{(1)}(k) = 0.5x^{(1)}(k-1) + 0.5x^{(1)}(k) \quad k=2,3,\dots,n; \quad (5)$$

Then we can get the grey difference equation of GM (1, 1) is:

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (6)$$

The method to find the values of a and b is usually use the least square method. We have

$$\theta = (B^T B)^{-1} B^T Y = \begin{bmatrix} a \\ b \end{bmatrix} :$$

$$\text{Where } Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2)1 \\ -z^{(1)}(3)1 \\ \vdots \\ -z^{(1)}(n)1 \end{bmatrix}$$

The  $\theta$  values represent the weight of comparative series to the referential series.

According to the solution of first differential equation, the initial value is  $x^{(0)}(1)$  and  $x^{(0)}(1) = x^{(1)}(1)$  from the general method, we can find that the discrete response of GM(1,1) model is as follows:

$$x^{(1)}(k+1) = (x^{(0)}(1) - b/a) \exp(-ak) + b/a \quad (7)$$

$$x^{(0)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k) \quad k=1,2,\dots,n \quad (8)$$

Wherein  $x^{(1)}(k+1)$  is the prediction value of  $x^{(1)}(k+1)$  and  $x^{(0)}(k+1)$  is the prediction value of  $x^{(0)}(k+1)$  at point  $k+1$ . The  $x^{(0)}(k+1)$  can also be obtained by using the inverse accumulated generating operation:

$$x^{(0)}(k+1) = (1 - \exp(a))(x^{(0)}(1) - b/a) \exp(-ak) \quad (9)$$

After the prediction value was get, the error of GM (1, 1) model is defined as follows:

$$e(k) = x^{(0)}(k) - x^{(0)}(k) / x^{(0)}(k) \times 100\% \quad (10)$$

In which,  $k=2,3,\dots,n$ ,  $x^{(0)}(k)$  indicates the actual value, and  $x^{(0)}(k)$  indicates the predicted value.

### 3. Results and Discussion

#### 3.1 Data Processing

In this paper, in order to examine whether GM (1, 1) model can fit the rate of inactivation with free chlorine for *Enterococcus faecalis* or not, we employ the experimental data of the rate of inactivation during the Ct value from 3.45 to 8.60  $\text{mg} \cdot \text{min} \cdot \text{L}^{-1}$ , it is shown in Table I.

Table I Comparison of Rate of Disinfection Prediction Accuracy

Ct value( $\text{mg} \cdot \text{min} \cdot \text{L}^{-1}$ )	Actual value of inactivation rate(log)	GM(1,1)of inactivation rate(log)	Relative Error(%)
3.45	1.38	1.38	0
3.90	1.53	1.53	0.04
5.60	1.54	1.92	9.75
7.20	2.63	2.42	7.94
7.20	2.91	3.04	4.62
8.00	4.07	3.83	6.05
8.60	4.73	4.82	1.76

### 3.2 Use Of GM(1,1) Model

The calculation steps for GM (1, 1) model are described briefly as follows:

$$X^{(0)}=(1.38,1.53,1.54,2.63,2.91,4.07,4.73)$$

An accumulated generating operation is used once on the original sequence, yielding the data sequence.

$$X^{(1)}=AGO X^{(0)}=(1.8,2.91,4.65,7.28,10.19,14.26,18.99)$$

The least square method is then used to derive the coefficients a and b, the values are

$$a=-0.230$$

$$b=1.041$$

The ordinary differential equation derivation method can derive the discrete  $x^{(1)}$  response equation value is as follows:

$x^{(0)}(k+1)=(1-\exp(-0.23))(x^{(0)}(1)-1.041/(-0.23))\exp(0.23k)$  In view of  $-a<0.3$ , the GM(1,1) model established can be used for long-term forecast.

The relative error test was then used to compare the actual value with predicted value, the computer calculations are provided in Figure 1. As shown in Table I, the mean relative error of the predicted values during the Ct value from 3.45 to 8.60 is 5.0244%. So we could obtain a conclusion that the GM(1,1) model could achieve the goal of predicting the rate of disinfection *Enterococcus faecalis* with free chlorine.

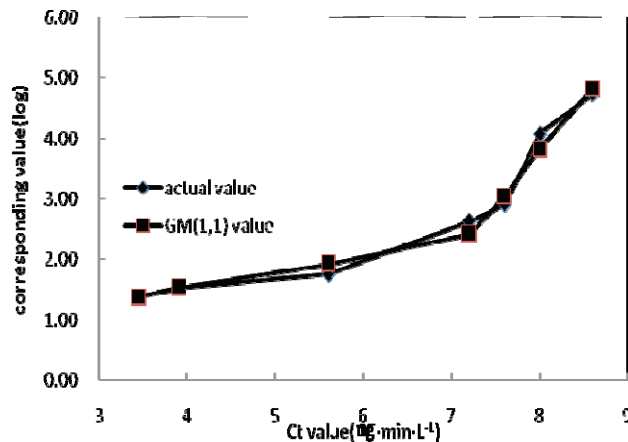


Fig.1 Comparison of rate of disinfection between prediction and actual value

### 4. Conclusions

The analysis performed in this paper suggests that the rate of inactivation with free chlorine for *Enterococcus faecalis* can be predicted using GM (1, 1) model, and the simulation results can be drawn as follows: the accuracy of prediction will be above 90%. This is identical with grey theory's ability to make prediction on the basis of lacking data and a small sample.

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